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CRITICAL VELOCITY OF GAS IN HEAT AND MASS TRANSFER EQUIPMENT

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A formula for calculating the critical gas velocity is found that is in good correlation with experimental data. Its generalization to the case of the presence of regular roughness on the surface of a packed bed is obtained.

In designing film heat and mass transfer equipment one should know with sufficient accuracy critical loads of gas and liquid phases in a packed bed. So that exchange processes may be more intense, loads of liquid and gas should be in the region of their strong hydrodynamic interaction. However, they should not be so large that intense droplet entrainment and flooding develop. The available theoretical and experimental data [1-3] differ in the values of critical loads by more than 200% (Fig. 1).

At small gradients of temperatures and concentrations for two-phase film flows the development of instability on the phase interface is caused mainly by the factors of the hydrodynamic interaction between the phases [1]. This occurs for a wide class of film heat and mass exchange equipment where the rate of thermal and diffusion processes is small (water towers, absorbers, film concentrators, distillers, air coolers, etc.).

In the present paper, within the framework of the linear theory of hydrodynamic stability in an isothermal approximation, a formula is obtained for calculating the critical gas velocity in a smooth cylindrical channel and, as a particular case, in a plane channel. Then this formula is put in correspondence with numerous experimental data and is generalized to the case of the presence of regular roughness on the working surface of a nozzle.

In an annular liquid film flow along the wall of a smooth cylindrical tube and gas flow, as shown in [4] on the basis of results of Miles [5] and Brook-Benjamin [6], for long waves the contribution of fluctuating tangential stress to the forces tending to change the shape of the wave on the film surface is much smaller than that of normal stress. On this basis, when studying the stability of the liquid film surface, one can consider both media (liquid and gas) as ideal fluids and apply the theory of potential flows. Then we determine a wave flow of liquid and gas in a vertical cylindrical tube from the solution of a boundary-value problem written in cylindrical coordinates (axis OX is oriented toward g) [2, 7-9]:

$$\frac{\partial^2 \varphi_i}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi_i}{\partial r}\right) = 0\,, \tag{1}$$

$$\frac{\partial \varphi_i}{\partial \tau} + \frac{1}{r}\left(\left(\frac{\partial \varphi_i}{\partial x}\right)^2 + \left(\frac{\partial \varphi_i}{\partial r}\right)^2\right) - gx + \frac{p_i}{\rho_i} = f(\tau)\,. \tag{2}$$

Here and below, $i = 1$ refers to liquid, 2 to gas.

At $r = R$

$$\frac{\partial \varphi_1}{\partial r} = 0\,, \tag{3}$$

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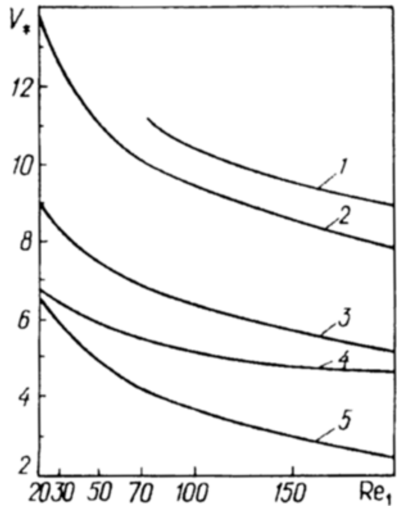


Fig. 1. Dependence of the critical gas velocity V_* (m/sec) on Re_1 according to data of different authors for a water-air system: 1) [2]; 2) [3]; 3) [2]; 4) [1]; 5) [2].

at $r = 0$

$$\frac{\partial \varphi_2}{\partial r} = 0 , \tag{4}$$

where $\varphi_i = \varphi_i(\tau, x, r)$ are the velocity potentials determined by the equalities $u_i = \partial\varphi_i/\partial x$, $v_i = \partial\varphi_i/\partial r$; the conjugation conditions on a free surface at $r = R - h_0 = \Delta$ are

$$\frac{\partial \varphi_1}{\partial r} = \frac{\partial h}{\partial \tau} + u_0 \frac{\partial h}{\partial x}, \tag{5}$$

$$\frac{\partial \varphi_2}{\partial r} = \frac{\partial h}{\partial \tau} + v_0 \frac{\partial h}{\partial x}, \tag{6}$$

$$p_1 - p_2 = \sigma \left(h''_{xx} - \frac{1}{h} \right) . \tag{7}$$

Let the equation of the phase interface have the form $h(\tau, x) = \Delta + \exp(ik(x - c\tau))$, where $c = c_1 + ic_2$. In accordance with this we seek the velocity potentials in the form

$$\varphi_1(\tau, x, r) = u_0 x + \psi_1(r) \exp(ik(x - c\tau)) , \tag{8}$$

$$\varphi_2(\tau, x, r) = v_0 x + \psi_2(r) \exp(ik(x - c\tau)) . \tag{9}$$

Having substituted (8) and (9) into Eq. (1) and using conditions (3)-(6), we have

$$\varphi_1(\tau, x, r) = u_0 x + \frac{ia(u_0 - c)\exp(ik(x - c\tau))}{I_1(k\Delta) - sK_1(k\Delta)} (I_0(kr) + sK_1(kr)) ,$$

where $s = I_1(kR)/K_1(kR)$, $I_0(r)$, $I_1(r)$, $K_1(r)$ are modified Bessel functions,

$$\varphi_2(\tau, x, r) = v_0 x + ia(v_0 - c) \frac{I_0(kr)}{I_1(k\Delta)} \exp(ik(x - c\tau)) .$$

The dispersion relation is found by eliminating p_1 and p_2 from (2), (7). As a result we have

$$A (u_0 - c)^2 - B (v_0 - c)^2 = \frac{\sigma}{k\Delta^2} - \sigma k , \tag{10}$$

where

$$A = \rho_1 \frac{K_1 (kR) I_0 (k\Delta) + I_1 (kR) K_0 (k\Delta)}{K_1 (kR) I_1 (k\Delta) - I_1 (kR) \; K_1 (k\Delta)} ; \quad B = \rho_2 \frac{I_0 (k\Delta)}{I_1 (k\Delta)} .$$

Allowing for the fact that $c = c_1 + ic_2$, we separate the real and imaginary parts in (10). Thus, we have the system

$$A ((u_0 - c_1)^2 - c_2^2) - B ((v_0 - c_1)^2 - c_2^2) = \frac{\sigma}{k\Delta^2} - \sigma k ,$$

$$c_1 = (Au_0 - Bv_0)/(A - B) ,$$

from which (on the basis of the condition $c_2 = 0$) we find the critical gas velocity V_*. The instability of the annular flow will start above it. We have

$$V_* = u_0 + \left(\frac{\sigma k}{AB} (A - B) \left(1 - \frac{1}{\Delta^2 k^2} \right) \right)^{1/2} .$$

For the counterflow, with allowance for the fact that $B/A << 1$, the critical velocity is

$$V_* = \left(\frac{\sigma k}{\rho_2} \frac{I_1 (k\Delta)}{I_0 (k\Delta)} \left(1 - \frac{1}{k^2 \Delta^2} \right) \right)^{1/2} - u_0 .$$

Assuming that the wave number $k = \zeta / h_0$ (ζ – const), we find

$$V_* = \left(\frac{\sigma \xi}{\rho_2 h_0} \frac{I_1 (\Delta\xi / h_0)}{I_0 (\Delta\xi / h_0)} \left(1 - \frac{h_0^2}{\xi^2 \Delta^2} \right) \right)^{1/2} - u_0 . \tag{11}$$

For a smooth plane channel $h_0/R << 1$. With allowance for this fact in Eq. (11), we have

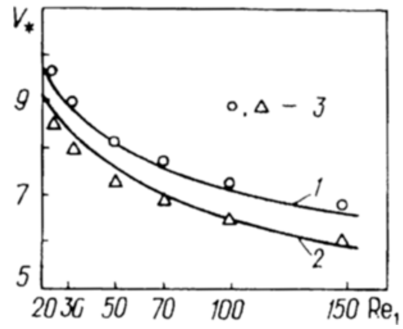
$$V_* = \left(\frac{\sigma \xi}{\rho_2 h_0} \text{th} \left(\xi \left(\frac{r}{h_0} - 1 \right) \right) \right)^{1/2} - u_0 .$$

In [1], as a result of generalization of extensive experimental studies a dimensionless relation was obtained that makes it possible to calculate critical loads of gaseous and liquid phases in a counterflow and allows for the length and diameter of the channel:

$$V_* = 1.346 \frac{\nu_2}{d} \text{Re}_1^{0.38} \text{We}^{0.113} \left(\frac{\rho_2}{\rho_1} \right)^{0.513} \left(\frac{\mu_1}{\mu_2} \right)^{0.455} \left(\frac{d}{h_0} \right)^{1.628} f(H, d) , \tag{12}$$

$$f(H, d) = \frac{H}{d} (0.38d - 0.015) + 0.07d^{-0.8} .$$

It is also shown that the relative root-mean-square deviation of test data from the computational relation (12) does not exceed 4.1%. A similar comparison with test data of other authors gave a deviation of 9.6%. In Eq. (11) the



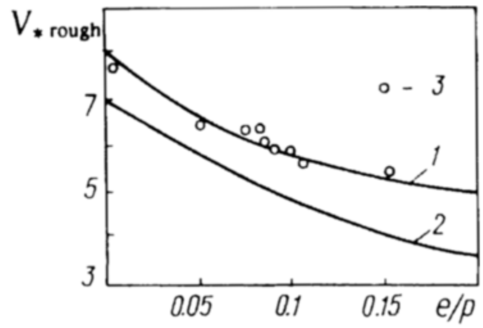


Fig. 2. Dependence of V_* (m/sec) on Re_1 for $H = 0.5$ m: 1) $d = 16$ mm; 2) 24, our data; 3) [1].

Fig. 3. Dependence of $V_{*\,rough}$ (m/sec) on the relative height of roughness protrusions e/p: 1) $H = 0.5$ m; 2) $H = 1.0$ m (our data); 3) [11].

value of ζ remained undetermined. We assign it, allowing for the viscosity of the media and the Weber number, in the form

$$\xi = \gamma\, \mathrm{We}^{a_1} \left(\frac{\mu_2}{\mu_1}\right)^{b_1},$$

where γ, a_1, b_1 are subject to determination. To allow for the effect of the height of the packed layer H, we write Eq. (11), following (12), in the form

$$V_* = \left(\left(\frac{\sigma\xi}{\rho_2 h_0}\,\frac{I_1(\xi\Delta/h_0)}{I_0(\xi\Delta/h_0)}\left(1 - \frac{h_0^2}{\xi^2\Delta^2}\right)\right)^{1/2} - u_0\right) f(H, d) \tag{13}$$

(12 mm < d < 38 mm). The constants in (13) are found by correlating relations (12) and (13). Using the least-squares method we obtain

$$\gamma = 0.01\,; \quad a_1 = -0.5\,; \quad b_1 = -0.12\,. \tag{14}$$

Figure 2 presents the dependence (13) obtained for the critical velocity in the case of a countercurrent flow of phases in a vertical channel with smooth walls and the corresponding data from [1].

Use of a regular roughness of the surface has gained wide acceptance as a method for enhancing heat and mass transfer [10]. The presence of regular roughness on a tube surface, when the height of the roughness protrusions is comparable to the mean thickness of the liquid film, leads to additional destabilization of the two-phase flow. Standing waves of large amplitude dominate on the film surface; the wave crest corresponds to the roughness protrusion. In strong hydraulic interaction the amplitudes of these waves tend to grow with time, which naturally results in a decrease in V_*. We seek the critical velocity in the presence of roughness in the form of a power dependence on the relative height of roughness protrusions:

$$V_{*\mathrm{rough}} = V_*\left(1 - a_2\left(\frac{e}{p}\right)^{b_2}\right), \tag{15}$$

where V_* is determined by relation (13), e, p are the height and period of regular roughness. Processing of experimental data of [11] gives the values

$$a_2 = 0.88\,, \quad b_2 = 0.572\,. \tag{16}$$

Relations (13)-(16) hold within the following ranges of the parameters: 20 < Re_1 < 100; 0.3 m < H < 0.85 m; 16 mm < d < 24 mm; $0 \le e/p < 0.2$. The deviation from test data does not exceed 10.1%. A dependence of the critical velocity of gas on the relative height of roughness protrusions is presented in Fig. 3.

The results obtained permit calculation of the maximum permissible velocity of a gas flow in film heat and mass transfer equipment with a densely packed bed [12, 13] and can easily be introduced into computer programs for designing and optimizing this equipment.

NOTATION

u_i, v_i, velocity components along the x, r axes, respectively, m/sec; p_i, pressure, Pa; ρ_i, density, kg/m^3; g, acceleration of gravity, m/sec^2; τ, time, sec; h, liquid film thickness, m; u_0, v_0, mean-flow-rate velocities of the liquid and gas, m/sec; σ, surface tension, N/m; R, tube radius, m; μ_i, coefficient of viscosity, N·sec/m^2; ν_i, coefficient of kinematic viscosity, m^2/sec; d, H, diameter and length of the tube and channel, m; $Re_l = h_0 u_0/\nu_l$, $We = \rho_l h_0 u_0^2/\sigma$, Reynolds and Weber numbers.

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